Beyond Anisotropy – Part II: Physical Models in LMR Space

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The following is the second part of the article “Beyond Anisotropy – Part 1: A Prestack Perspective” (RECORDEr, 35-7). This section provides an in depth analysis of the seismic reflection of anisotropic media, specifically using the Lambda-Mu-Rho (LMR) crossplot. In Part 1, anisotropic reflections were analyzed from the perspective of zero-offset P and S reflectivities and Thomsen’s (1986) anisotropic parameters. The concept of a zero-gradient line was defined and the different AVO classes of Rutherford and Williams (1989) were plotted on P and S reflectivity crossplot, referred to as the AVO circle.

In Part 2, the anisotropic reflection is analyzed from the perspective of a Lambda-Mu-Rho (LMR) crossplot. This allows for an intuitive feel of how reflections will change with variations in lithology, porosity or fluid filling in addition with the presence of anisotropic media. Subsequently, four distinct anisotropic AVO classes are introduced each relating to the zero-gradient line concept. Finally, the physical implications of anisotropy as it relates to fractures, is related to the LMR crossplot and basic interpretation templates are introduced.

Physical Significance

The anisotropic reflection of interest describes Horizontal Transverse Isotropy (HTI) and is given by Ruger (1997) as

\[
R_{p}^{HTI}(\theta) = \frac{1}{2} \frac{\Delta Z}{Z} + \left[ \frac{\Delta V_{p}}{V_{p}} - \frac{2V_{s0}}{V_{p0}} \right]^{2} \frac{\Delta G}{G} + \left( \frac{V_{s0}}{V_{p0}} \right) \Delta \gamma \cos^{2} \phi \sin^{2} \theta + ...
\]

To ascribe some physical significance to the anisotropic responses described in Part 1, the reflectivity crossplot (AVO circle) is transferred to the Lambda-Mu-Rho (LMR) crossplot. The LMR crossplot is a way of comparing the incompressibility and rigidity of any rock. It can be computed using compressional and shear logs through the following equations,

\[
\begin{align*}
\lambda p &= I_{p}^{2} - 2I_{S}^{2} \\
\mu p &= I_{S}^{2}
\end{align*}
\]

Transferring P and S reflectivity to an LMR crossplot is done by identifying lines of constant P and S impedance. With lines of constant P and S impedance, it is possible to compute lines of constant P and S reflectivity with respect to the P and S impedance of the overlying layer. The assumption of negligible density reflectivity is made so that

\[
\begin{align*}
\frac{\Delta I_{p}}{I_{p}} &\approx \frac{\Delta V_{p}}{V_{p}} \\
\frac{\Delta I_{S}}{I_{S}} &\approx \left( \frac{\Delta G}{G} \right)^{1/2}
\end{align*}
\]

Knowing the overlying layer, IP1 and IS1, it is an arithmetic exercise to compute the P and S zero-offset reflectivities through

\[
\begin{align*}
\frac{\Delta V_{p}}{V_{p}} &= \frac{2(I_{p2} - I_{p1})}{(I_{p2} + I_{p1})} \\
\frac{\Delta G}{G} &= \frac{2(I_{s2}^{2} - I_{s1}^{2})}{(I_{s2}^{2} + I_{s1}^{2})}
\end{align*}
\]

In part one, the zero-gradient line was computed for Ruger’s VTI and HTI equation. The zero-gradient lines can be computed for both the isotropic and anisotropic cases on an LMR crossplot for a given set of anisotropic parameters.

\[
\begin{align*}
\frac{\Delta G}{G} &= -\frac{1}{4} \left( \frac{V_{p0}}{V_{s0}} \right)^{2} \frac{\Delta V_{p}}{V_{p0}} + \frac{1}{4} \left( \frac{V_{p0}}{V_{s0}} \right)^{2} \Delta \delta \\
\frac{\Delta G}{G} &= \frac{1}{4} \left( \frac{V_{p0}}{V_{s0}} \right)^{2} \frac{\Delta V_{p}}{V_{p0}} + \frac{1}{4} \left( \frac{V_{p0}}{V_{s0}} \right)^{2} \Delta \delta^{(v)} + 2 \Delta \gamma
\end{align*}
\]

Plotting these on an LMR crossplot allows for an intuitive sense of what seismic reflections can be expected.

In figure 1, the blue lines are lines of constant P impedance, while the green lines are lines of constant shear impedance. The lines of constant P and S impedance also correspond to lines of constant reflectivity with respect to the overlying layer, denoted by the black square. The orange and pink curves are the zero-gradient lines for the isotropic and anisotropic (HTI) AVO, respectively. The red line has the property that P and S reflectivity are equal (Rp=Rs). With these curves the AVO reflectivity circle can be superimposed on an LMR crossplot and distinct AVO types can be identified. The position of the overlying layer (black square) defines the intersection of zero-gradient line (isotropic or anisotropic) and the Rp=Rs line. With these two lines, the five common AVO types can be identified. Type 1 is bounded by the Rp=Rs line, the boundary between Vp/Vs increases and decreases. The zero P impedance reflectivity line defines Type
2 while Type 3 is identified by zero-gradient line and decreases in P impedance, relative to the overlying layer. Type 4 AVO is the zero-gradient line. Type 5 occupies the space between the zero-gradient and Rp=Rs line; a positive gradient, a decrease in Vp/Vs ratio and a decrease in P impedance.

The following table shows three examples, also used in Part 1, to illustrate the effects of anisotropy on an LMR crossplot.

<table>
<thead>
<tr>
<th>Example</th>
<th>Layer 1</th>
<th>Layer 2</th>
<th>Vp (km/s)</th>
<th>Vs (km/s)</th>
<th>Rp (km/s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Example 1</td>
<td>2.39</td>
<td>2.42</td>
<td>4000</td>
<td>4350</td>
<td>2800</td>
</tr>
<tr>
<td>Example 2</td>
<td>2.45</td>
<td>2.41</td>
<td>4000</td>
<td>3500</td>
<td>2400</td>
</tr>
<tr>
<td>Example 3</td>
<td>2.41</td>
<td>2.39</td>
<td>4000</td>
<td>3500</td>
<td>2250</td>
</tr>
</tbody>
</table>

Table 1. Seismic properties of layers used to construct full 3 term synthetic amplitude responses as a function of offset and azimuth.

Consider example 1

As shown by the AVO circle, the decreases in Vp/Vs ratio are the seismic signatures of interest. In figure 2, the green square represents the overlying layer while the circle is the reservoir layer of interest. In the parallel to fracture direction, the isotropic zero-gradient line is used to determine the AVO type and it indicates a Type 1 AVO response with a large negative gradient. The AVO circle does not show any new information but superimposed on the LMR crossplot allows for integration of the AVO response in conjunction with rock properties providing more insight into the seismic signature. For example, in this case the green line represents a Hashin-Shtrikman (HS) rock model for a 75% quartz, 25% clay and an Sw=0.25. Layer 2 on the LMR crossplot would correspond to a 7% porosity rock of this type. Moving down the HS line towards the origin of the LMR crossplot represents increases in porosity. One can predict the amount of porosity required to produce AVO Types 2 through 5 given the same overlying layer.

In the cross fracture direction, the HTI zero-gradient line moves to overlay the reservoir layer indicating a zero-gradient, Type 4, AVO response. The AVO circle moves to accommodate the new zero-gradient and Rp=Rs lines. The new Rp=Rs line exists to account for the new position of layer 1. The intersection of these two lines defines where properties of layer 1 would exist to produce an isotropic equivalent AVO response. In this case, as seen in figure3, the result is a false Type 1 because although it has a negative gradient, it represents an increase in Vp/Vs ratio. Note that the HS line now indicates that much smaller porosity variation is required to produce AVO types 2 through 5.

Examples 2 and 3 are seen in figure 4-5 and 6-7 respectively. Example 2 fits an 85% quartz 15% clay and an Sw=0.25 with a porosity of 12% while example 3 fits a 65% quartz 35% clay and an Sw=0.25. In addition, from Bakulin et al. (2000), the zero-gradient line expressions can be converted to accommodate the physical models of Hudson (1981) and Schoenberg and Sayers Linear Slip Theory (1995). Under the assumption of weak anisotropy, Schoenberg and Sayers normal and tangential fracture weaknesses express Ruger’s HTI anisotropic parameters as

\[
\begin{align*}
\epsilon^{(v)} &= -2 \frac{V_S^2}{V_P^2} \left( 1 - \frac{V_S^2}{V_P^2} \right) \Delta_N \\
\delta^{(v)} &= -2 \frac{V_S^2}{V_P^2} \left( 1 - 2 \frac{V_S^2}{V_P^2} \right) \Delta_N + \Delta_T \\
\gamma^{(v)} &= -\frac{\Delta_T}{2}
\end{align*}
\]
Figure 3. AVO circle on LMR crossplot for example 1 in the cross fracture direction.

Figure 4. AVO circle on LMR crossplot for example 2 in the parallel to fracture direction.

Figure 5. AVO circle on LMR crossplot for example 2 in the cross fracture direction. The position of the HTI zero-gradient line acts to decrease the gradient in the cross fracture direction relative to the parallel to fracture direction.
Hudson’s model reduces to the following for gas-filled and fluid-filled cracks respectively

\[
\varepsilon^{(v)} = -\frac{8}{3} \varepsilon \\
\delta^{(v)} = -\frac{8}{3} \left[ 1 + \frac{V_S^2}{V_P^2} \left( \frac{1}{3} - \frac{2V_S^2}{V_P^2} \right) \right] \\
\gamma^{(v)} = -\frac{8}{3} \left( \frac{3 - 2 \frac{V_S^2}{V_P^2}}{3 - 2 \frac{V_S^2}{V_P^2}} \right) \\
\varepsilon^{(v)} = 0 \\
\delta^{(v)} = -\frac{32 V_S^2}{V_P^2} \frac{\varepsilon}{3 \left(3 - 2 \frac{V_S^2}{V_P^2}\right)} \\
\gamma^{(v)} = -\frac{8}{3} \left( \frac{3 - 2 \frac{V_S^2}{V_P^2}}{3 - 2 \frac{V_S^2}{V_P^2}} \right) 
\]
Here, \( e \) is the crack density. The zero-gradient line can then be expressed, assuming weak anisotropy and an isotropic overlying layer, in terms of normal and tangential fracture weaknesses combining equations 6b and 7:

\[
\frac{\Delta G}{G} = \frac{1}{4} \frac{V_p^2}{V_s^2} \frac{\Delta V_p}{V_p} + \Delta_N \left( \frac{V_s^2}{V_p^2} \frac{1}{2} \right) + \frac{1}{2} \Delta_T \tag{9}
\]

The zero-gradient line represented by crack density, for gas-filled and fluid-filled cracks, respectively, can be expressed as

\[
\frac{\Delta G}{G} = \frac{1}{4} \frac{V_p^2}{V_s^2} \frac{\Delta V_p}{V_p} + \left( 2 \left( 3 + 12 \frac{V_s^2}{V_p^2} - 8 \left( \frac{V_s^2}{V_p^2} \right)^2 \right) e \right)
\]

\[
\frac{\Delta G}{G} = \frac{1}{4} \frac{V_p^2}{V_s^2} \frac{\Delta V_p}{V_p} + \frac{8}{3} \left( 3 - 2 \frac{V_s^2}{V_p^2} \right) \left( 1 - \frac{V_s^2}{V_p^2} \right) e \tag{10}
\]

These physical models allow for a more in-depth understanding of how the change in gradient with azimuth is related to fracture weakness or crack density.

To highlight the use of the LMR crossplot consider the zero-gradient line from a fracture weakness perspective where the anisotropic term is

\[
\Delta_N \left( \frac{V_s^2}{V_p^2} \frac{1}{2} \right) + \frac{1}{2} \Delta_T \tag{11}
\]

In fact, with a standard \( V_p/V_s \) ratio of 2, the anisotropic term simplifies to

\[
\frac{1}{2} \left( - \frac{1}{2} \Delta_N + \Delta_T \right) \tag{12}
\]

It is noticed that when \( \Delta_T > 1/2 \Delta_N \), the anisotropic term is positive and thus the zero-gradient line will move towards positive shear reflectivities. When \( \Delta_T < 1/2 \Delta_N \), the anisotropic term is negative and the zero-gradient line will move towards negative shear reflectivities. Thus, when dealing with positive reflectivities (increases in \( R_p \) and \( R_s \) across interface) and tangential weakness is greater than half the normal weakness (positive anisotropy), one should expect a decreasing gradient when moving azimuthally towards the cross fracture direction. If the tangential weakness is less than half the normal weakness (negative anisotropy), the same reflectivity pair the gradient should increase in the cross fracture direction. However, for a negative reflectivity pair (decreases in \( R_p \) and \( R_s \) across interface), the opposite trends will hold. With tangential weakness greater than half the normal weakness, gradient will increase in the cross fracture direction, while with tangential weakness less than half the normal weakness the gradient will decrease in the cross fracture directions.

From the perspective of Hudson’s penny shaped crack model, the zero-gradient line is defined by the crack density, \( e \). In a gas filled crack case, with a standard \( V_p/V_s \) ratio of 2, the anisotropic term reduces to \(-32/45e\) where for the fluid filled case the anisotropic term reduces to \(16/15e\). In general, it is expected that the gas filled cracks will have negative anisotropy while the fluid filled cracks will have positive anisotropy. Thus depending on the reflectivity pair, the gradient may increase or decrease. However, once the reflectivity pair has been identified, the gradient polarity will be a clear indication of gas versus fluid fill, according to Hudson’s model. Note that for fluid filled cases, it is reasonable to assume that \( \Delta_N \approx 0 \) (Bakulin et al. 2000) and as such, from equation 11, the anisotropic term should always be positive. In dry cases, with \( \Delta_N > \Delta_T \) the expectation is of negative anisotropic terms. This agrees with Hudson’s model.

In general, as summarized in figure 8, if the isotropic gradient term is accounted for, the residual anisotropic gradient will be of:

- **Class A**: positive when magnitude of \( \Delta(\delta) < 8(V_s/V_p)^2\Delta \gamma \)
- **Class B**: negative when magnitude of \( \Delta(\delta) > 8(V_s/V_p)^2\Delta \gamma \)
- **Class C**: zero when magnitude of \( \Delta(\delta) = 8(V_s/V_p)^2\Delta \gamma \)

As pointed out by Goodway et al (2010), there is gradient ambiguity in Ruger’s HTI equation. Because \( \delta \) and \( \gamma \) are of opposite signs, there exist combinations where the anisotropic terms will
be zero, i.e. Class C when \( \Delta \delta^{(V)} = 8(V_s/V_p)^2\Delta \gamma \). In such cases, the higher order terms neglected earlier play an important role. The higher order terms can influence the gradient in ways that are counter to what was just outlined. As such, care must be taken when assessing azimuthally varying gradients and assuming the zero-gradient line’s position on an LMR cross-plot. No variation in azimuth could imply a variety of scenarios, with isotropy being just one of the possible explanations. The expected anisotropic response from forward modeling can help to resolve many potential ambiguities.

For the special case of elliptical anisotropy, there exists a crossover angle where for a given angle of incidence there will be no variation of amplitude with azimuth. This angle is given by

\[
\cos^2 \theta = \frac{V_F^2}{V_s^2} \frac{\Delta \delta^{(e)}}{8\Delta \gamma} \tag{13}
\]

This is the angle where the higher order terms cause a change in gradient and will be referred to as Class D anisotropic reflections. Note that this will only occur when \( \Delta \delta < 8(V_s/V_p)^2\Delta \gamma \), a consequence of equation 13; \( \cos^2(\theta) \) must be positive and less than 1. In essence, Class D is a specific subset of a Class A anisotropic AVO response. Consider Table 2 and figure 9 which is an example of Class D anisotropic AVO. Consider Table 2 and figure 9 which is an example of Class D anisotropic AVO. From equation 13, the crossover angle for this example is calculated to be 25 degrees. Note that the crossover angle is not the angle at which the amplitude changes from positive to negative but where the gradient is dominated by the higher order terms.

The definition of the four classes can be converted to normal and tangential weaknesses or crack density using equations 7 and 8.
fracture weakness perspective, Class A would imply that $\Delta_l$ is much larger than $\Delta_N$, which can be interpreted to suggest fluid filled cracks. Class B would imply $\Delta_N$ is much larger than $\Delta_l$, which can be interpreted to suggest gas filled cracks. Figure 11 illustrates the concepts for both A and B Class anisotropic reflections and their interpretations.

Note that Class C anisotropic signatures will not display any differences between the parallel and cross fracture direction inversions. Figure 12 shows Class A and B anisotropic reflections and their definitions from three different perspectives: Thomsen’s parameters, fracture weaknesses and Hudson’s crack density model. In addition, the interpreted gas and fluid effects are labeled.

**Conclusion**

Determining the zero-gradient lines in isotropic media allows for an understanding of AVO classes with knowledge of P and S reflectivity. The additional knowledge of anisotropic parameters also allows for predictability of azimuthally varying gradients in HTI media. Transferring this information onto LMR crossplot allows for further insight as the seismic reflection is immediately related to rock properties. Four distinct anisotropic classes for HTI media are introduced: Class A when once the isotropic gradient term is removed the residual anisotropic gradient is positive $-\Delta D^V < (8(V_S/V_P)^2A_l$ – Class B when the residual anisotropic gradient is negative $\Delta D^V > (8(V_S/V_P)^2A_l$ – Class C when $\Delta D^V = (8(V_S/V_P)^2A_l$ – and Class D, a subset of Class A where with elliptical anisotropy there is a gradient change. In all instances, forward modelling of anisotropic AVO response is recommended to understand the true nature of the seismic signal. 

**References**


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